**Electron-Electron Interaction (Nearly Free)**

Now we’re going to try to add in the e-e interaction, and see what effects it might be responsible for. So let’s go back to our H – no disorder.



where the electron number density is:



where i is the ith electron’s position (operator). We’ll go back to the approximation that the ions are immobile. Then we’d just have:



We’ll treat the e-ion interaction and ion-ion interaction in the continuum approximation, presuming all the interactions are the same, and taking ni(r) = ne(r). One problem with doing this, however, is that we effectively lost all information about band structure, and all information about how filling affects the properties of the substance. For instance, if we had a completely filled band, then we should get no transport properties, etc., but we have no way of ‘completely filling’ our band here. We can certainly specify the mobile electron density to be what it would take to fill the band, but our nearly free model will presume a strictly parabolic free particle energy spectrum, i.e., not take into account band structure, and so we can’t expect to get any information about what happens for nearly filled bands. This is one advantage of the tight-binding model discussed later. But this analysis should apply pretty well to nearly free electrons, like those found in s (Group 1, 2) or p (Group 13) subshells.



Reprising our work in the 2nd quantization file, the KE part is:



as we’d expect. And the interaction part reads:



but then look at the q = 0 term,



it just cancels out. So we can say,



So our Hamiltonian comes to,



We will need the Fourier transform of the two particle potential. In 3D the result has already been calculated.



(presuming q is slightly imaginary to aid convergence) In 2D, we’d have (with the help of a convergence factor)



If we use faux Gaussian units, then we’d do ε0 → 1/4π.

**Feynman Rules**

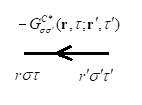
Well we basically already did this in the QM and SM folders. But I’ll repeat, for the case of calculating the complex time single particle GF.



So,

**Real Space Rules**

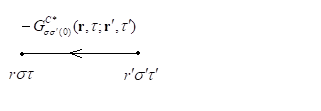
So we have just one GF now



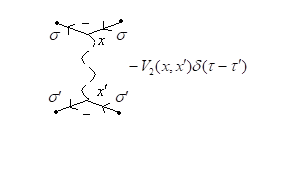
and the external points are:



which we connect with the bare GF:



via powers of our interaction vertex:



Then the general procedure is as follows…

**Topology**

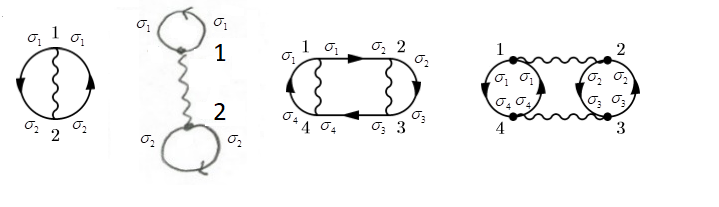
Connect all topologically distinct, fully connected (meaning all end points must be connected to each other and no vaccum bubbles), diagrams together, associating each element in diagram with the indicated term. Note the order of indices is important, Gσσ´(xk,xj) ≠ Gσ´σ(xj,xk), so always order the positions against the directions of the arrows.

**Equal time issues**

There are two equal time issues, depicted below. As usual, should interpret Gσσ´(x,t;x´,t) as Gσσ´(x,t;x´,t+), and in particular Gσσ(x,t;x,t) as Gσσ(x,t;x,t+) = -<ψσ(x,t)ψ†σ(x,t+) > = <ψ†σ(x,t)ψσ(x,t) > = <nσ(x)> which is the thermal average of the density.

**Signs/Numerical Factors**

Each diagram is associated with a numerical factor, because each topologically distinct diagram comes from a variety of contractions and because we must factor in the 1/n! term coming from the S-matrix expansion. Basically, diagrams with external legs seem to have a simple net factor of unity. Bubbles have a symmetry factor of 1/p, where p is the number of permutations (including the identity permutation) of points that result in the same diagram [note discussion below presumes all vertices are *same* sign]. For instance,



We can do the permutation (12) and so the first guy gets a factor of ½. Same with the second guy. The third gets a factor of ½ as well because all we can do is the permutation (13)(24). Can verify that flipping diagram about horizontal and vertical would result in this permutation and so it works. Not sure why (14)(23) wouldn’t also be acceptable, but for the possible fact that there doesn’t seem to be a corresponding set of reflections that would result in this permutation, like there was for the previous. Then the last guy has the permutations, (12)(34), (13)(24), and (14)(23). So it gets a factor of ¼.

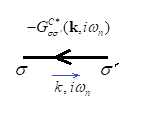
(Fermion) loops have their own special sign factors. Each fermion loop gets a factor of (-ε), which would be just 1 for a boson loop, -1 for a fermion loop – this has to do with intracacies of transposing the operators to put them in proper order. The four diagrams have 1, 2, 1, 2 Fermion loops respectively.

**Sum**

Then integrate/sum over all internal vertices, including spin ones. Apropos the spin sum, since it’s conserved by H and G0σσis diagonal in spin index, then we’ll note that if we start off trying to calculate G↑↑, then the δσσ´ attached to our GF, coupled with our spin sum, will simply have the effect of enforcing all other GF’s in the diagram to also be G0↑↑. Likewise if we tried to calculate the spin down GF. So basically we can ignore the spin sum part. Except in one instance: fermion loops. In these parts of the diagram, the GF’s are in some sense disconnected from the external points, and since they circle back on themselves we end up with a term like Σσ1,σ2,σ3G(0)σ1σ2G(0)σ2σ3G(0)σ3σ1. And this reduces to Σσ3G(0)σ3σ3G(0)σ3σ3G(0)σ3σ3 = G(0)↑↑G(0)↑↑G(0)↑↑ + G(0)↓↓G(0)↓↓G(0)↓↓ = 2[G(0)↑↑G(0)↑↑G(0)↑↑], say. And so we just need to multiply our diagrams by 2 (or 2s+1 if higher spin) to account for these loops. The simple factor of two *does* presume that G(0)↑↑ = G(0)↓↓, however, and it will be in our case.

**Fourier Space Rules**

So we have our single GF again:



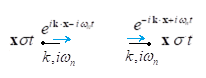
where these G’s are the Fourier transforms of the real space guys,



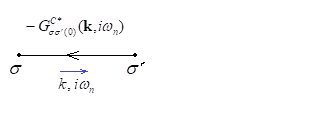
We connect the external points,



These carry the Fourier transform variable. And if we’re ultimately interested in the real space, time GF, then we must add the following factors to each:



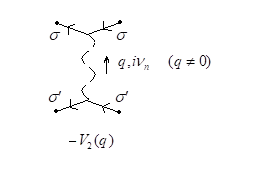
We connect these with the bare GF:



where,



via powers of the bare impurity vertex:



(If potential line has just q = 0 term on it, then it’s zero) We still have iνn flowing down the line because of the artificial time-dependence brought in by the δ(t-t´) we put on each interaction potential. Note it’s a bosonic frequency, though, as energy conservation demands, because we cannot have electron lines with fermionic frequencies if we have a fermionic interaction frequency traveling into it (EM interaction is carried by bosons too – is this a coincidence?).

**Topology**

Connect all topologically distinct, fully connected (meaning no vacuum bubbles), diagrams together, associating with each element in the diagram the indicated term. Remember that all energy-momentum labels **k**, ω must be going in the same way w/r to the GF arrow. They must all be going against the arrows (or with them I suppose). And then we conserve energy/momentum at each vertex.

**Equal time issues**

Apropos the equal time issue, in Fourier space this would be handled as:



So we need to include a factor of exp(iωn0+) for each bare Green’s function closed in on itself. Fundamentally, the problem occurs when the temporal argument of the bare Green’s function is zero.

**Signs/Numerical Factors**

Same as above.

**Sum**

Then sum/integrate over all independent momenta - wavenumbers/energies-frequencies/indices. If finite spatial transform, then the guy on the left. If continuous spatial transform, then the guy on the right:



and include the external frequency/wavenumber if want the real space GF.